How do we determine if a quadrilateral is a parallelogram?

If each pair of opposite sides on a quadrilateral are parallel, then, by definition, the quadrilateral is a parallelogram.

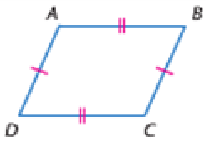
However, this is not the only test to prove if a quadrilateral is a parallelogram:

Theorems Conditions for Parallelograms

6.9 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Abbreviation *If both pairs of opp. sides are ≅, then quad. is a ▱.*

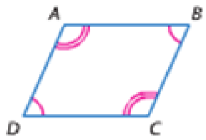
Example If $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$, then $ABCD$ is a parallelogram.



6.10 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Abbreviation *If both pairs of opp. ∠s are ≅, then quad. is a ▱.*

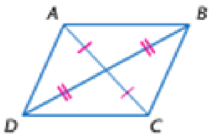
Example If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.



6.11 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Abbreviation *If diag. bisect each other, then quad. is a ▱.*

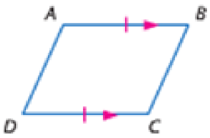
Example If $\overline{AC}$ and $\overline{DB}$ bisect each other, then $ABCD$ is a parallelogram.



6.12 If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.

Abbreviation *If one pair of opp. sides is ≅ and ||, then the quad. is a ▱.*

Example If $\overline{AB} \parallel \overline{DC}$ and $\overline{AB} \cong \overline{DC}$, then $ABCD$ is a parallelogram.



In list form:

We have 7 properties of Parallelograms.

We have 5 tests for parallelograms.

Concept Summary

Prove that a Quadrilateral Is a Parallelogram

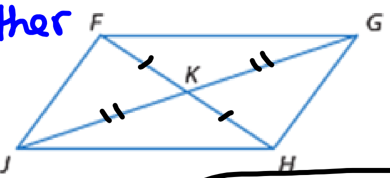
- Show that both pairs of opposite sides are parallel. (Definition)
- Show that both pairs of opposite sides are congruent. (Theorem 6.9)
- Show that both pairs of opposite angles are congruent. (Theorem 6.10)
- Show that the diagonals bisect each other. (Theorem 6.11)
- Show that a pair of opposite sides is both parallel and congruent. (Theorem 6.12)

prove

Example 1:

Test: diagonals bisect each other

If $FK = 3x - 1$, $KG = 4y + 3$, $JK = 6y - 2$, and $KH = 2x + 3$, find x and y so that the quadrilateral is a parallelogram.



Set FK=KH:

$3x-1=2x+3$
$-2x+1 \quad -2x+1$

$x=4$

FK= $3(4)-1=11$ ✓
KH= $2(4)+3=11$

Set JK=KG:

$6y-2=4y+3$
$-4y+2 \quad -4y+2$

$\frac{2y}{2}=\frac{5}{2}$

$y=2.5$

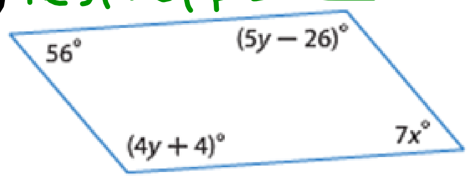
JK= $6(2.5)-2=13$ ✓
KG= $4(2.5)+3=13$

FGHJ is a parallelogram when $x=4$ and $y=2.5$.

Example 2 and 3:

Find x and y so that each quadrilateral is a parallelogram.

2) Test: opp. $\angle$s $\cong$



To get x:

$\frac{56}{7} = \frac{7x}{7}$

$8 = x$

$7(8) = 56$ ✓

To get y:

$5y - 26 = 4y + 4$

$-4y + 26 \quad -4y + 26$

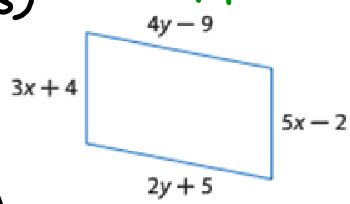
$y = 30$

$5(30) - 26 = 124$ ✓

$4(30) + 4 = 124$

It's a parallelogram when $x = 8$ and $y = 30$.

3) Test: opp. sides $\cong$



To get x:

$5x - 2 = 3x + 4$

$-3x + 2 \quad -3x + 2$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$

$5(3) - 2 = 13$

$3(3) + 4 = 13$ ✓

To get y:

$4y - 9 = 2y + 5$

$-2y + 9 \quad -2y + 9$

$\frac{2y}{2} = \frac{14}{2}$

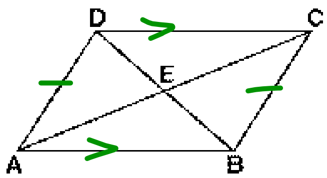
$y = 7$

$4(7) - 9 = 19$

$2(7) + 5 = 19$ ✓

It's a parallogram when $x = 3$ and $y = 7$.

Given: Quadrilateral ABCD below



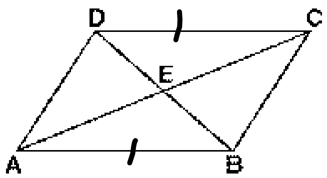
4) If $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$, determine whether quadrilateral ABCD is a parallelogram. [*Explain*.]

Yes – both pairs of opp. sides $\cong$.

5) If $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \cong \overline{BC}$, determine whether quadrilateral ABCD is a parallelogram. [*Explain*.]

NO – need same pair of sides $\parallel$ and $\cong$.

Given: Quadrilateral ABCD below



6) If AE = EC and DE = EB, determine whether quadrilateral ABCD is a parallelogram. [*Explain*.]

Yes - diagonals bisect each other.

7) If $\overline{DC} \parallel \overline{AB}$, determine whether quadrilateral ABCD is a parallelogram. [*Explain*.]

No - need both pairs of opp. sides $\parallel$.

8) If $\overline{DC} \cong \overline{AB}$, determine whether quadrilateral ABCD is a parallelogram. [*Explain*.]

No - need both pairs of opp. sides $\cong$.